GOING FROM "I DON'T LIKE MATH" TO "MATH ISN'T *THAT* BAD"

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High stakes testing and the rush to calculus have made algebra competence the benchmark for our math curriculum. What about students who don't meet with academic success in algebra? An inability to factor polynomials does not preclude a student from learning about bigger math ideas like algorithms. Come see how Graph Theory, Number Theory, and Cryptography can provide realistic and understandable opportunities for students to engage in rich mathematics regardless of their algebra ability level.

BACKGROUND / CONTEXT

It began with this email from the math department chair. My first year at a new school and I was already complaining about one of the courses I was teaching – Transition to College Math.

Originally this course was conceived as a bold attempt to offer mathematically challenged seniors a chance at improving their mathematical understanding and giving them the opportunity of adding a fourth year of math to their schedule. The belief at the time was that the success of this new course would depend largely on implementing alternative instructional methods rather than the traditional lecture format.

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-----Original Message-----
Sent: Friday, November 05, 1999 2:12 PM
Subject: Transition

     You really have cart blanche on structuring the
class any way you see fit. This might be a perfect
opportunity for you to experiment with some novel ideas
that you have on math education. I know that the
response was positive about Excel.

     I have never seen it as a review class. I would
rather the course took up topics like matrices and
finance and probability and solving techniques than
merely revisiting old topics. These would be topics
that they would encounter in a finite math class at the
college level which most of them will have to take.
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With the focus placed on changing teaching methodology, discussion of curriculum content was a non-issue. Since almost all of these mathematically challenged seniors had earned below a B minus in Algebra 2 (thus not eligible to enroll in Pre Calculus), it seemed natural to focus the curriculum on reinforcing and extending fundamental geometry and algebra topics since these were the students' forté (many of these students had earned poor grades in geometry and algebra).

Not long into the first semester, I got the sense that neither the students nor I were enjoying the course. Motivation – not student intelligence – was the issue. The students did not seem interested in math or worse, disliked math. While teaching methodology is a factor for student motivation, I started to think the curriculum played a greater role. Many of these students had already taken courses in both geometry and algebra but had not achieved academic success. It was also a good bet that the only mathematics they had ever been presented was rooted in geometry and algebra. Maybe it was time for the curriculum to shift towards different types of mathematics.

So what was the result? Transition to College Math morphed into a course called Finite Math which then was renamed to Discrete Math (finite apparently didn't sound like a good thing) and over the last 7 years, I have been cobbling together an ever-growing binder of activities, challenges, and resources organizing these into three categories: Graph Theory, Number Theory, and Cryptography. Why these? (1)They are very different from geometry and algebra. (2)They play a role in many tangible, easy to understand real-world scenarios. (3)I enjoy these topics!

OBSERVATIONS / RESULTS

- While working without a textbook is liberating, it can quickly lead to sloppiness and/or aimlessness
- Not all students embrace the absence of "drill and practice"
- Many teachers are intimidated by the mathematics involved in the course
- Students enjoy the cryptography component of the course the most

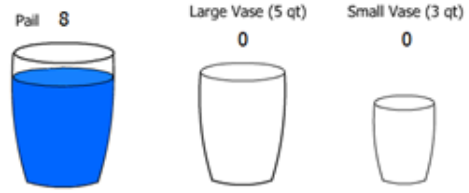
ALGORITHMS, QUANTIFIABLE DIFFERENCES, & MODELING

❶ Class Notes/Activity

Using Graph Theory to Model Systems

The Water Puzzle

Instructions: Starting with a pail containing 8 quarts of water and small (3-quarts full) and large (5-quarts full) vases as measuring devices, you are to split the water in the pail equally between two people. Thus there must be 4 quarts of water in the pail and 4 quarts of water in the 5 quart vase at the end of the puzzle.



What is the *minimum* number of pours required to achieve the goal of 4 quarts of water in each pail?

source of this activity: http://webspace.ship.edu/deensley/DiscreteMath/flash/ch7/sec7_5/waterpuzzle.html

❷ Activity

Efficiency Algorithms in Graph Theory

Suppose you are a T.A. for the Dean's Office and have been assigned to drop off walkie-talkies on Wednesdays before lunch. The rooms on your delivery list are always the same: 103M, 217C, 112, and Campus Ministry. Because you dislike excess walking and wasting time, you want to find the most efficient route possible. How do you do that?

❸ Assignment

Using Graph Theory to Model Systems

Our math department has 10 classes that need to be scheduled in the spring. Below is a list of classes that cannot be scheduled at the same time due to teacher or enrollment constraints.

Class	Cannot be Scheduled During the Same Period as
A	D, I
B	D, I, J
C	E, F, I
D	A, B, F
E	C, H, I
F	C, D, I
G	J
H	E, I, J
I	A, B, C, E, F, H
J	B, G, H

Task

In the space below, draw a graph that represents the information in the table that will help you determine the minimum number of periods required to schedule every class.

source of this activity: http://www.colorado.edu/education/DMP

ALGORITHMS, PATTERNS, & PREDICTING

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Class Notes

Prime Numbers

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140

4031	4032	4033	4034	4035	4036	4037	4038	4039	4040
3561	3562	3563	3564	3565	3566	3567	3568	3569	3570
2071	2072	2073	2074	2075	2076	2077	2078	2079	2080
1671	1672	1673	1674	1675	1676	1677	1678	1679	1680

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Class Notes & Activity

Modular Arithmetic & Predicting the Future

JOE'S DINER

Joe's Diner has become your favorite lunch spot because the food is soooo good! In fact, you have made it a point to bring clients to Joe's Diner for lunchtime meetings. Joe has 4 different lunch specials (see chart below) that he offers on a continuously rotating basis (i.e. special #1, special #2, special #3, special #4, special #1, special #2, special #3, etc.). Joe begins every year with special #1 – so every January 1st special #1 is offered. Joe is also a workhorse and never closes – his restaurant is open every day of every year! Below is a listing of the specials.

Special #1:	Garden Salad, Baked Ham, Green Beans & Mashed Potatoes
Special #2:	Ceasar Salad, Pineapple Chicken, Carrots & Snow Peas
Special #3:	Garden Salad, Savory Meatloaf, Asparagus & Garlic New Potatoes
Special #4:	Ceasar Salad, Pine Nut Salmon, Rice Pilaf & Broccoli

Can You Predict the Future?

What will be the special offered on February 3rd?

What kind of salad will be included on the special for April 20th?

You have a lunch meeting with a new client on October 25th and want to take the client to Joe's for a business lunch. However, you have found out that your new client is following a strict Atkins Diet – no carbs! Will you be able to offer one of Joe's oh so tasty specials to your client or should you reschedule your lunch meeting date?

ALGORITHMS & PATTERNS

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Class Notes

Getting Rid of Charts by Using Modular Arithmetic

Affine Ciphers

An ***affine cipher*** is a system that utilizes both addition and multiplication in a modular arithmetic system. To encode or decode a message you need to know the two ***keys*** – the additive key and the multiplicative key. While there is no restriction on the additive key, the multiplicative key must be ***relatively prime*** to the mod you are using. Why??

Suppose that you have an affine cipher with additive key 8 and multiplicative key 5. What letter would replace the letter 'D'? Since 'D' is the 4th letter of the alphabet, the mathematical computation would go as follows:

$$D \Rightarrow 4 \Rightarrow \underbrace{4+8=12}_{\text{additive step}} \Rightarrow \underbrace{12\cdot 5=60}_{\text{multiplicative step}} \Rightarrow \underbrace{60\equiv 8 \pmod{26}}_{\text{equivalence step}} \Rightarrow 8 \Rightarrow H$$

So the letter 'D' would be replaced with the 8th letter of the alphabet, namely 'H', since the initial number 4 was changed into the number 8 using a combination of addition and multiplication in modulo 26 arithmetic. GOODBYE CHARTS!

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PLANNING (November 24-30)

As a group you are to develop an Action Plan for this project. This means discussing and deciding as a group who is going to be responsible for the various parts of the project, establishing deadlines, identifying strategies that will be followed, how the presentation will be organized, etc. The action plan should as descriptive as possible and show evidence of significant, reflective thought. Consulting the **grading rubric** is an important part of the planning phase.

PRODUCTION (December 1-7)

You will be given class time during which you will have access to computers, the internet, and library resources. During this class time your group is expected to work productively on the required components of the project. Your group is expected to communicate daily via your group's Discussion Board. During the project there are *benchmarks* that your group must meet to show the progress of your work. Most of these *benchmarks* consist of updating files to your group's Documents. Specific benchmark deadlines are found at right.

PRESENTATION (December 8-9)

Members of each group will showcase their work by making a brief (2 to 3 minute) "sales" presentation highlighting the features of their Cipher System. Along with the presentation, each group will provide the class with a "product speficication sheet" that gives details on how their Cipher System works by providing a step-by-step enciphering and deciphering example. A student who is not in class on the scheduled presentation day will have those points deducted from *his* assessment score.

CHALLENGE (December 9-10)

Groups will be matched against each other to test the security of their Cipher System. This is an opportunity for groups to obtain extra points towards their overall project grade.

Benchmark Deadlines

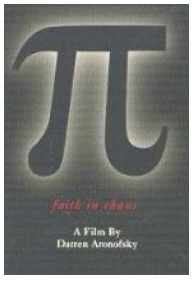
Tuesday 25 November (9pm)
Inaugural Action Plan
Tuesday 02 December (9pm)
Inaugural Spreadsheet & Powerpoint
Wednesday 03 December (9pm)
Updated Action Plan
Thursday 04 December (9pm)
Updated Spreadsheet & PowerPoint
Sunday 07 December (9pm)
Final Spreadsheet, PowerPoint & Action Plan

Assessment

Group projects will be assessed using the following **grading rubric** -- it consists of 7 areas for which the group can earn between 0 and 2 points. The sum of the points earned will be multiplied by a factor of 3 resulting in the maximum points possible being 75 (which will be approximately 25% of your semester grade). In extreme cases, the Action Plan may be used to adjust the points for individual group members should it become apparent that an individual did not contribute his efforts towards the project.

AN UNUSUAL END

During the final week of the course, I show the movie ***Pi : Faith in Chaos*** by Darren Aronofsky (he's better known for *Requiem for a Dream*). It is unusual, not the easiest film to watch, and often is mathematically flawed (as chronicled on Wikipedia).



So why do I show this movie? Because of the richness of mathematical discussion it can spur in relation to the course – one of the movie's central themes is about finding and describing patterns! The movie brings up topics like the Golden Ratio/Spiral, Kabala, and the number pi all in the context of a surreal(?) connection to the divine. Wow!