WAYS TO NATURALLY REVISIT GEOMETRY PROOFS IN ALGEBRA CLASS

André Mathurin, Mathematics Teacher at Bellarmine College Preparatory (San Jose, CA)
amathurin@bcp.org **http://webs.bcp.org/sites/amathurin**

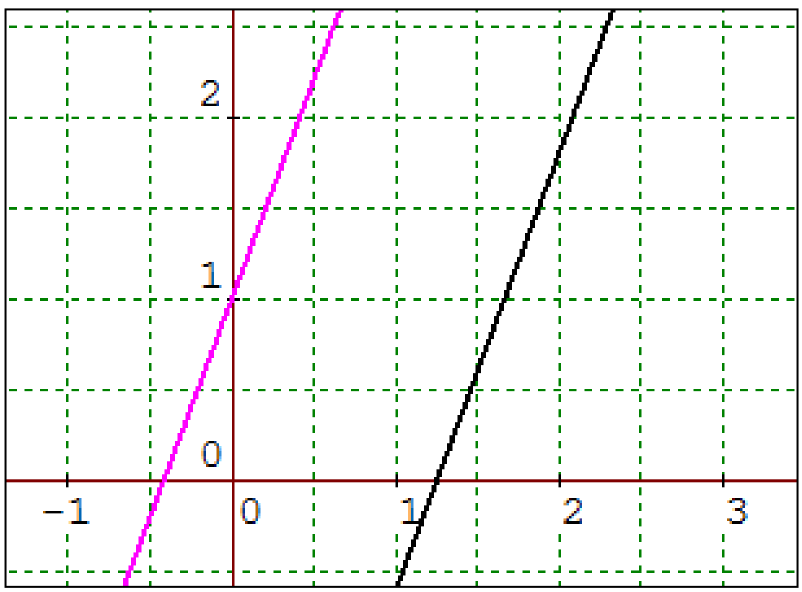
❶ HOW DO YOU *KNOW* THE LINES PERPENDICULAR?

Most proficient algebra students can tell you that if two lines have the same slope, then the lines are parallel. But if pressed to explain why two lines that have the same slope implies that the lines are parallel, chances are the proficient algebra student's reply will involve a circular tangle of equivocations (i.e. "because parallel means the lines have the same slope") or an overly– generalized explanation (i.e. "because having the same slope means the lines are going the same direction"). Even math tutorial websites sometimes fall prey to these difficulties:

> Parallel lines and their slopes are easy. Since slope is a measure of the angle of a line from the horizontal, and since parallel lines must have the same angle, then parallel lines have the same slope — and lines with the same slope are parallel.

How could you provide a formal, mathematically solid explanation while at the same time inter-relating algebra and geometry concepts? Let's explore the relationship between the lines { EMBED Equation.DSMT4 } and { EMBED Equation.DSMT4 }

Let's look at the graph of the two lines is provided below. Choose a point on the *x*–axis to the right of each line's *x*–intercept. With this point, construct a right triangle using a portion of the graph of each line as the hypotenuse and the segment along the *x*–axis defined by the line's *x*–intercept and the point you chose.



Are the two right triangles you formed congruent? Maybe, but not necessarily. Is there any definite relationship between the two right triangles you formed? They are similar. Why is that?

What do we know about similar triangles? Their corresponding angles are congruent. So the angles formed by the *x*–axis and the graphs of the lines are congruent. The *x*–axis acts as a transversal and thus we have two lines cut by a transversal with a pair of corresponding angles congruent. This implies that the lines are parallel. We started with the lines having the same slope and concluded with the lines are parallel.

How could you provide a similar proof for two lines with slopes whose product equals negative one resulting in the two lines being perpendicular? **(*see separate handout*)**

❷ SLOPES, SYSTEMS OF EQUATIONS, MIDPOINT, & CIRCLES

While most students will remember that three non-collinear points define a plane, they might be less familiar with the fact that through any three non-collinear points a circle can be defined. Let's consider the following:

> ***Find the equation of the circle that passes through the points { EMBED Equation.DSMT4 }, { EMBED Equation.DSMT4 }, and { EMBED Equation.DSMT4 }***

There's a good chance students might be unsure about how to approach this question. And here is where revisiting geometry proofs will become valuable for helping develop and guide the process.

The algebraic equation of a circle depends upon two elements, namely the location of the center and the measure of the radius. But, since the measure of the radius is also dependent on the location of the center, our first goal should be to find the center of the circle. Time for a revisit to geometry…

What pencil and instrument method(s) are available for finding the center of a given circle?

(*see separate handout***)**

Armed with the proof that the perpendicular bisector of a chord contains the center of a circle, we can utilize it as a plan of attack for the original question.

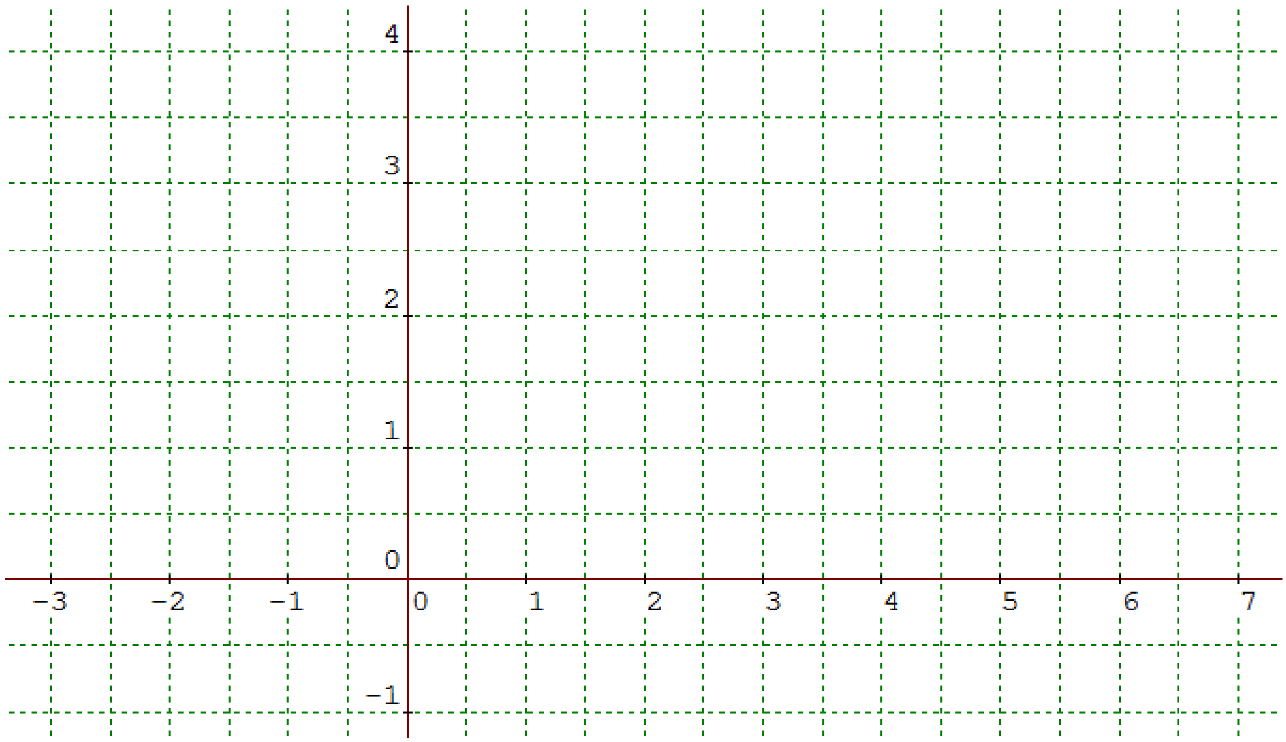
❸ FURTHER CONNECTION POINTS

I believe that by providing opportunities for students to make connections between algebraic and geometric representations helps enhance a students' overall understanding of mathematics. The challenge is to find useful junctions in the curriculum that naturally lend themselves to these types of discussions or activities. Some ideas for other possible areas that could be developed include the following:

- Suppose you want to make a triangle where two of the sides measure 6 and 20. If you randomly choose a measure for the third side of the triangle from the set of measures that would allow the triangle to exist, what is the probability that the resulting triangle will be an obtuse triangle? right triangle? acute triangle?
- Using algebraic equations to develop a method for generating Pythagorean Triples.
- Using matrices to compute the area of triangles.
- Creating an irrational ruler using the Pythagorean Theorem and right triangles as a way of visualizing the non-linear distribution of square roots on the number line as well as to explore to measure square roots

❶ **Given:** Two lines with slopes whose product is equal to negative one.

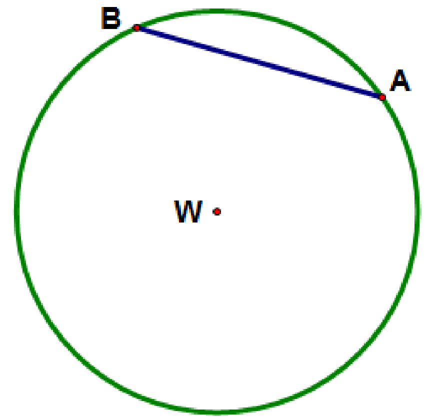
Prove: The two lines are perpendicular.



❷

Given: Circle W with cord AB

Prove: The perpendicular bisector of chord AB contains W



STATEMENTS	REASONS