GRAPH THEORY

❶ Animal Survival [Visual Representations, Equivalence, Appearance vs. Structure]

source of this activity: http://www.colorado.edu/education/DMP

The zoo keeper of a major zoo wants to redo the zoo in such a way that the animals live together in their natural habitat. Unfortunately, it is not possible to put all the animals together in one location because some are predators of others. The X marks in the chart at right show a predator-prey relationship, so those pair of animals cannot be safely placed in the same location.

	A	B	C	D	E	F	G	H
A		X			X			
B	X			X			X	
C								X
D		X				X		
E	X							
F				X				
G		X						
H			X					

Create a graph that represents the relationships indicated in the chart.

Connection/Extension Ideas

- What is the minimum number of locations required to safely house all of the animals?
- If the graph represented a computer network, what are the most crucial edges?

❷ Sprouts [Quantifiable Differences, Data Collection & Analysis, Pattern Recognition]

The Game of Sprouts was invented in 1967 by Princeton mathematician John H. Conway and by Michael S. Paterson, when both were at the University of Cambridge in the UK.

1. **With a partner, play a couple of Sprouts game that begins with 3, 4, or 5 vertices. While you play each game, collect data about the play of the game.**
2. **Based on the games you played and the data you collected, determine the maximum number of moves possible in a Sprouts game that begins with 42 vertices.**

Connection/Extension Ideas

- How could you figure out if there is a winning strategy?
- How are Sprouts games connected to 3-dimenional nets and Euler?

❸ Efficient Travel [Counting, Developing & Evaluating Strategies/Algorithms]

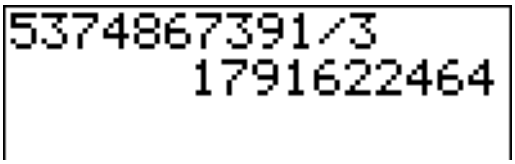
You are planning your holiday visit with family in who live in five different cities. Below is a matrix of the names and distances between these five cities.

	1	2	3	4	5
1 Tampa FL		10	27	6	15
2 Seffner FL	10		30	15	6
3 Clearwater FL	27	30		22	30
4 Lutz FL	6	15	22		19
5 Brandon FL	15	6	30	19	

1. **If you plan to stay with your family in Lutz, in what order should you visit each of the other cities in order to minimize your travelling distance?**
2. **If you wanted to achieve the absolute minimum traveling distance for visiting each city, where should your plan to stay?**

❹ Deceptive Calculator [Partitions, Divisibility Rules, Number System, Limits of Technology]

Below is a screen capture from a division calculation done using a TI-84 calculator.



Discuss at least two different ways that you can show that the calculator is providing false information.

Connection/Extension Ideas

- Is there a divisibility rule for multiples of 7?
- What are some algorithms for determining if a number is prime or composite?

❺ Unlucky 13 [Organizing Patterns, Modular Arithmetic, Taming the Infinite, Proof]

1. **What is the maximum number of times Friday the 13th that can occur within a single January to December calendar year? Show/Explain your method.**
2. **Is it possible to not have a Friday the 13th within a single January to December calendar year? Show/Explain your method.**

Connection/Extension Ideas

- How can you tell if a book ISBN number is valid or not?
- How do you deal with negative numbers in a modular system?

❻ Unusual Endings [Organizing Patterns, Modular Arithmetic, Taming the Infinite, Proof]

Step #1: Using the last two digits of your phone number, create a two digit number.
Step #2: If the number is even, then divide it by 2 and record the result. If the number is odd, then multiply it by 3 and add 1 and record the result.
Step #3: With the new number you just wrote down, repeat step #2.

Example:

30 → since 30 is even, 30 ÷ 2 = 15
15 → since 15 is odd, (3 × 15) + 1 = 46
46 → since 46 is even, 46 ÷ 2 = 23
23 → since 23 is odd, (3 × 23) + 1 = 70
⋮ ⋮

1. **How long could you continue this procedure?**
2. **What do you think would happen if you continued doing this?**

Connection/Extension Ideas

- How can you tell if a book ISBN number is valid or not?
- How do you deal with negative numbers in a modular system?

❼ Sneaky Scrambling [Identifying Patterns, Algorithms]

The letters in the phrase "What is Discrete Math" have been scrambled and placed in groups of three.

Scramble #1:	H T A	T E M	E R C	D I S	S I T	W H A
Scramble #2:	W E T	H R E	A C M	T S A	I I T	S D H
Scramble #3:	S T E	D M A	I T T	H A W	R E I	S C H
Scramble #4:	A H W	T M E	I A T	S T E	D H R	I S C

One of the scrambles was generated by randomly selecting the message letters from a hat while the other three scrambles were generated using an algorithm based on a basic geometrical concept.

1. **Identify which is the random scramble and explain how/why you arrived at your decision.**
2. **What geometrical concept was central to the scrambling algorithm?**

Connection/Extension Ideas

- How many total possible ways are there for scrambling the letters in the phrase?
- What are some modifications could you make to the scrambling algorithm?

❽ Sneaky Substitution [Modular Arithmetic, Functions, Combinatorics, Backwards is Harder]

Below is a scheme for replacing plaintext with ciphertext by scrambling the alphabet using modular arithmetic.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B	G	L	Q	V
	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2	7	12	17	22

1. **Write the word "SECRET" using the Modular Scramble ciphertext.**
2. **What is the pattern that the modular scramble follows?**
3. **We've received the secret message shown at right. What does it mean? It's now your job to reverse the process and decode.**

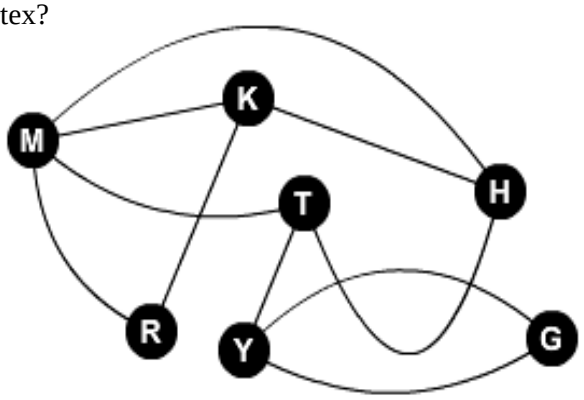
B L S M Y

Connection/Extension Ideas

- How can you use matrix algebra in conjunction with modular arithmetic to encipher/decipher?
- Create your own cipher system!

Sample Assessment Questions

1. In the graph shown at right, which of the following vertices is a cut vertex?
   A. K **B.** T **C.** H



2. The graph shown at right is planar.
   A. True **B.** False **C.** depends

3. The graph shown at right is not complete. How many more edges need to be ***drawn*** to make the graph complete?.
   A. 11 **B.** 21 **C.** 32

4. The graph shown at right does not contain an Euler path. Which of the following edges, when added to the graph, will allow an Euler path?
   A. RY **B.** MG **C.** YK **D.** RG

5. The number 86 is congruent to the number 50 in a modular 7 arithmetic system.
   A. True **B.** False **C.** Depends **D.** Impossible to tell

6. Which of the following is not a triangular number?
   A. 6 **B.** 24 **C.** 15 **D.** 36

7. ___ proved that $n^p \equiv 1 \pmod{p}$ whenever p is prime.
   A. Gauss **B.** Fermat **C.** Goldbach **D.** Euclid

8. Identify **<u>all</u>** (there is more than one) of the following number pairs that ***are not*** twin primes.
   (A) 521 & 523 (B) 59 & 61 (C) 67 & 71 (D) 87 & 89 (E) 859 & 861

9. Which of the following are characteristics of an effective cipher system:
   A. the enciphering process must be kept secret to be effective
   B. it must use both transposition and poly-graphic substitution
   C. the enciphering process must be reversible
   D. it must be complex to the point that it will be impossible for enemy to break
   E. enciphering/deciphering can be accomplished relatively quickly

10. Using only addition & multiplication, solve $27 + 3x \equiv 5 \pmod{10}$

11. Provide mathematical proof/evidence that the number 1,949 is prime.

12. Kevin decided to simplify his fashion life by devising a systematic way for choosing his daily attire. Starting January 1st of each year, he will wear the items listed first in the chart at right. On each subsequent day, he will wear the next item at the list. When he gets to the end of a particular list, he will move back to the top of that list. ***Use modular arithmetic*** to predict Kevin's attire on the 4th of July.

Pants/Shorts	Shirt	Shoes
Cargo Pants	Red Polo Shirt	Doc Martins
Acid-Washed Jeans	Brown T-Shirt	Sandals
Khaki Pants	Green Dress Shirt	Skechers
Paratrooper Shorts	Blue Thermal Shirt	
	Gray T-Shirt	
	Black Sweatshirt	

Jan	Feb	Mar	Apr	May	Jun
31	28	31	30	31	30

20. Prime numbers are important enough that people today spent time trying to find the next new prime numbers. Discuss some reasons why prime numbers are important.