Engaging Activities & Ideas for Teaching Discrete Math

http://tinyurl.com/amathurin-NWMC2013

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Step #1: Using the last two digits of your phone number, create a two digit number.

Step #2: If the number is even, then divide it by 2 and record the result.
If the number is odd, then multiply it by 3 and add 1 and record the result.

Step #3: With the new number you just wrote down, repeat step #2.

Example:

30 → since 30 is even, $30 \div 2 = 15$

15 → since 15 is odd, $(3 \times 15) + 1 = 46$

46 → since 46 is even, $46 \div 2 = 23$

23 → since 23 is odd, $(3 \times 23) + 1 = 70$

⋮ ⋮

❶ ANIMAL SURVIVAL

BIG IDEAS: Visual Representations, Equivalence, Appearance vs. Structure

source of this activity: http://www.colorado.edu/education/DMP

The zoo keeper of a major zoo wants to redo the zoo in such a way that the animals live together in their natural habitat. Unfortunately, it is not possible to put all the animals together in one location because some are predators of others. The X marks in the chart at right show a predator-prey relationship, so those pair of animals cannot be safely placed in the same location.

Create a graph that represents the relationships indicated in the chart.

	A	B	C	D	E	F	G	H
A		X			X			
B	X			X			X	
C								X
D		X				X		
E	X							
F				X				
G		X						
H			X					

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MATHEMATICAL ASSOCIATION OF AMERICA

Calculus

A Joint Position Statement of the Mathematical Association of America and the National Council of Teachers of Mathematics

Question: How should secondary schools and colleges envision calculus as the course that sits astride the transition from secondary to postsecondary mathematics for most students heading into mathematically intensive careers?

MAA/NCTM Position

Although calculus can play an important role in secondary school, the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. The college curriculum should offer students an experience that is new and engaging, broadening their understanding of the world of mathematics while strengthening their mastery of tools that they will need if they choose to pursue a mathematically intensive discipline.

Preliminaries

- Goals
- Format
- Background/Context

Preliminaries

- Goals
  - ✓ *Spark Interest in Exploring More*
  - ✓ *Highlight “Big-Ticket” Mathematical Ideas*
- Format
- Background/Context

Preliminaries

- Goals
- Format
  - ✓ *Summarize Activity & Mathematical Significance*
  - ✓ *Discuss Variations for Implementation*
- Background/Context

Preliminaries

- ✷ Goals
- ✷ Format
- ✷ Background/Context
  - ✓ *Evolution, Relevance, & Challenges*
  - ✓ *Meaningful & Timely*

CCSS.Math.Practice.MP1	**Make sense of problems and persevere in solving them.**
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A		X			X			
B	X			X			X	
C								X
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F				X				
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G		X						
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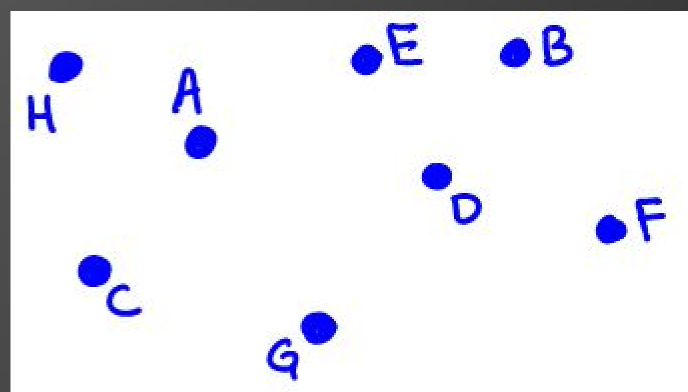

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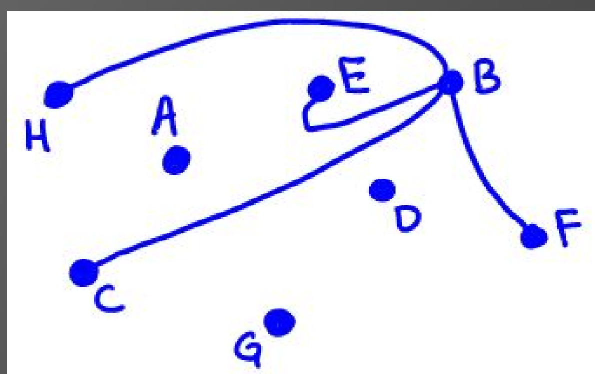
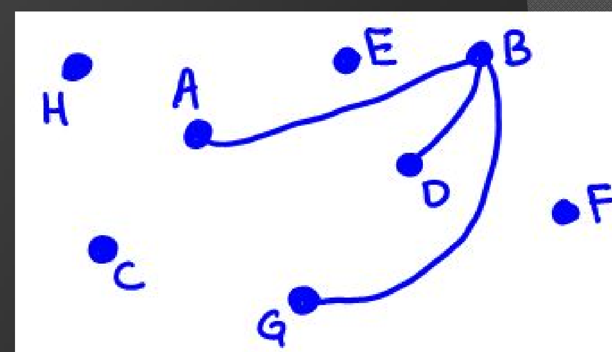
H			X					



What's the Difference?



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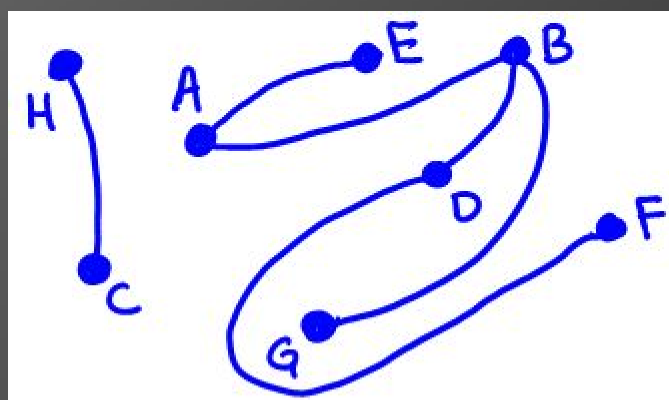

Is Your Graph the Same as My Graph?

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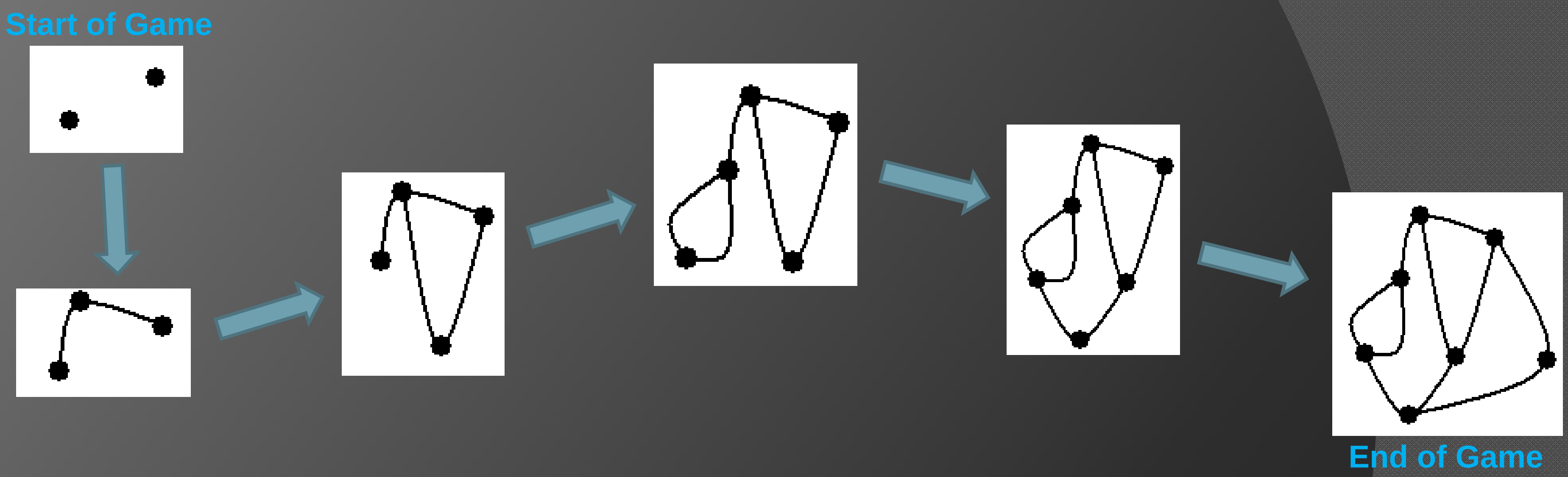
Connection/Extension Ideas

- What is the minimum number of locations required to safely house all of the animals?
- If the graph represented a computer network, what are the most crucial edges?

❷ SPROUTS

BIG IDEAS: Quantifiable Differences, Data Collection & Analysis, Pattern Recognition

The Game of Sprouts was invented in 1967 by Princeton mathematician John H. Conway and by Michael S. Paterson, when both were at the University of Cambridge in the UK.



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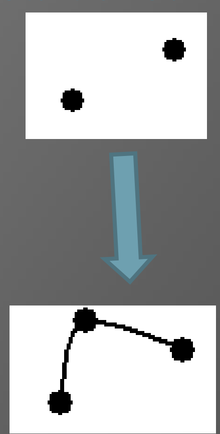
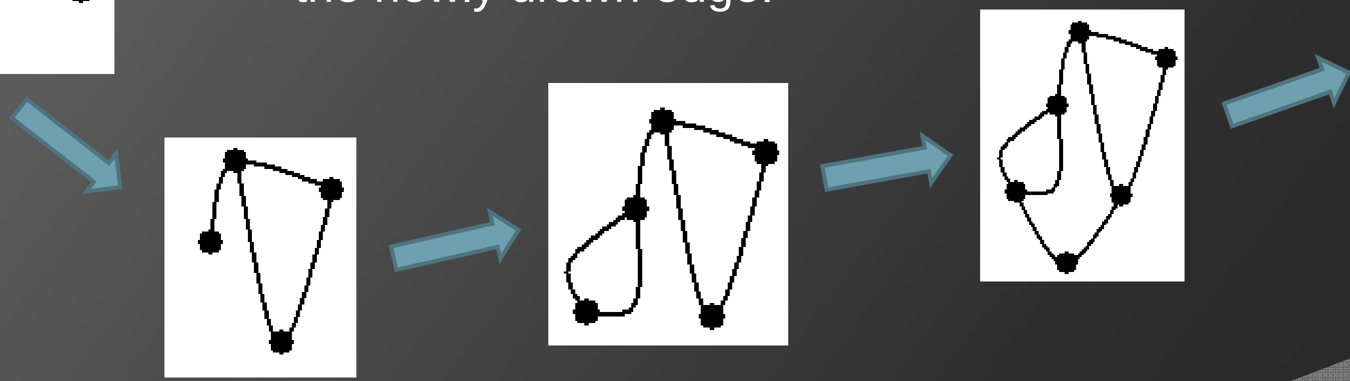
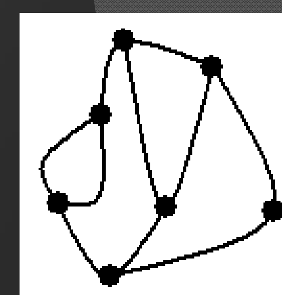
Rules for Playing Sprouts

1. The winner is the player who makes the last move.
2. No vertex have more than 3 edges.
3. A move starts by drawing an edge such that:
   - the edge starts and ends at a vertex
   - the edge does not cross an existing edge
4. The move ends by placing a new vertex along the newly drawn edge.

Start of Game







End of Game

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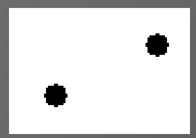
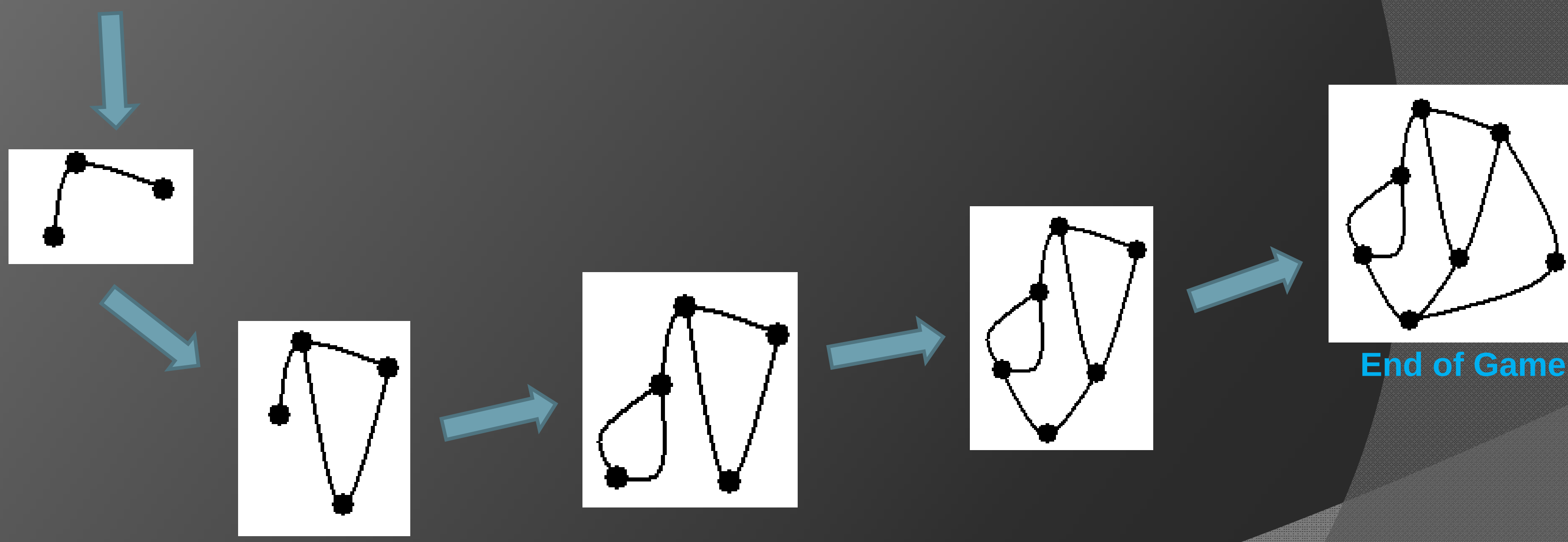
Determine the maximum number of moves possible in a Sprouts game that begins with 42 vertices. (without actually playing a game with that many vertices)

Start of Game





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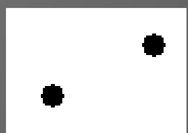

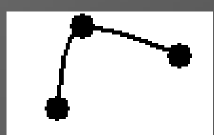

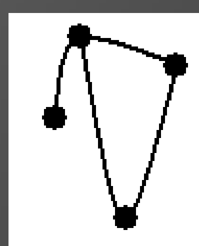

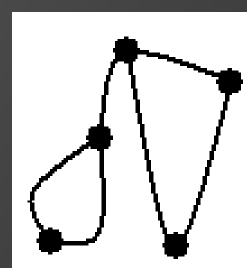

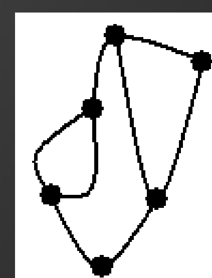

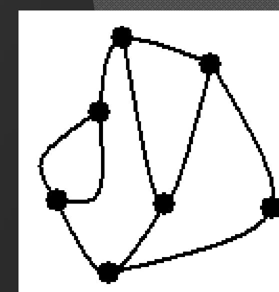

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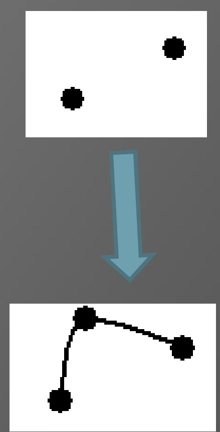
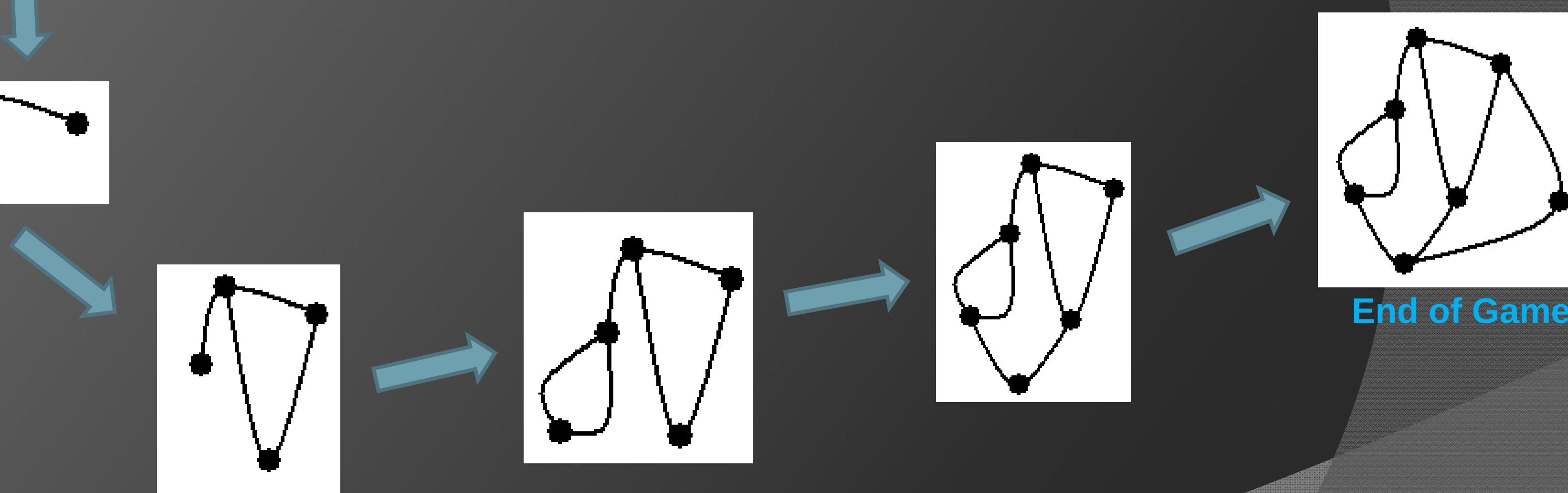
Move Number	Number of Vertices	The Degree of Each Vertex (separate each using a comma)	Sum of all Degrees	Number of Edges
0	2	0, 0	0	0
1	3	1, 1, 2	4	2
2	4	1, 2, 2, 3	8	4

Start of Game





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0	2	0, 0	0	0
1	3	1, 1, 2	4	2
2	4	1, 2, 2, 3	8	4
3	5	2, 2, 2, 3, 3	12	6
4	6	2, 2, 3, 3, 3, 3	16	8
5	7	2, 3, 3, 3, 3, 3, 3	20	10

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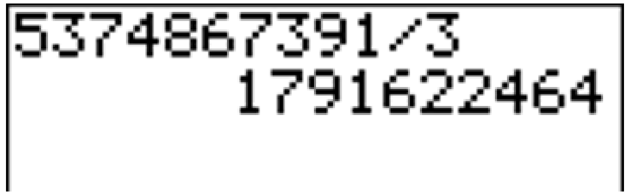
Connection/Extension Ideas

- How could you figure out if there is a winning strategy?
- How are Sprouts games connected to 3-dimenional nets and Euler?

❹ DECEPTIVE CALCULATOR

BIG IDEAS: Partitions, Divisibility Rules, Number System, Limits of Technology

Below is a screen capture from a division calculation done using a TI-84 calculator.



Discuss at least two different ways that you can show that the calculator is providing false information.

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```
5374867391/3
      1791622464
```

Discuss at least two different ways that you can show that the calculator is providing false information.

$$\begin{array}{r} 1,791,622,464 \\ \times \qquad\qquad 3 \\ \hline \ldots 392 \end{array}$$

Algorithmic Argument

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Partitioning Argument

$$5+3+7+4+8+6+7+3+9+1=43$$

43 is not divisible by 3

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```
5374867391/3
          1791622464
```

Discuss at least two different ways that you can show that the calculator is providing false information.

$$
\begin{aligned}
472 &= 400 + 70 + 2 \\
&= 4(100) + 7(10) + 2(1) \\
&= 4(99+1) + 7(9+1) + 2(1) \\
&= 4(99) + 4(1) + 7(9) + 4(1) + 2(1) \\
&= 4(99) + 7(9) + 4(1) + 7(1) + 2(1)
\end{aligned}
$$

$$5+3+7+4+8+6+7+3+9+1=43$$

43 is not divisible by 3

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Connection/Extension Ideas

- Is there a divisibility rule for multiples of 7?
- What are some algorithms for determining if a number is prime or composite?

❺ UNLUCKY 13

BIG IDEAS: Organizing Patterns, Modular Arithmetic, Taming the Infinite, Proof

What is the maximum number of times Friday the 13th that can occur within a single January to December calendar year? Show/Explain your method.

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Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
1	2	3	4	5	6	7
8	9	10	11	12	13	14

Mon	Tue	Wed	Thu	Fri	Sat	Sun
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

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				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
⋮	⋮	⋮	⋮	⋮	⋮	⋮
			42	43	44	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
			70	71	72	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
			98	99	100	101
102	103					
⋮	⋮	⋮	⋮	⋮	⋮	⋮

The 13th of	Corresponding Day of Year
JAN	13
FEB	31 + 13 = 44
MAR	28 + 44 = 72
APR	31 + 72 = 103
MAY	⋮
JUN	⋮
JUL	⋮
AUG	⋮
SEP	
OCT	
NOV	

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The 13th of	Corresponding Day of Year	Partitions Based on 7	Modular Form	Same Day of the week as
JAN	13	$13=1(7)+6$	$13\equiv 6 \pmod 7$	6th
FEB	31 + 13 = 44	$44=6(7)+2$	$44\equiv 2 \pmod 7$	2nd
MAR	28 + 44 = 72	$72=10(7)+2$	$72\equiv 2 \pmod 7$	2nd
APR	31 + 72 = 103	$103=14(7)+5$	$103\equiv 5 \pmod 7$	5th
MAY	⋮	⋮	⋮	⋮

How can this chart help answer the original question?

Is there a better way to answer the original question?

Doh! What about leap years??

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Connection/Extension Ideas

- How can you tell if a book ISBN number is valid or not?
- How do you deal with negative numbers in a modular system?

❼ SNEAKY SCRAMBLING

BIG IDEAS: Identifying Patterns, Making Connections, Algorithms

The letters in the phrase "What is Discrete Math" have been scrambled and placed in groups of three.

Scramble #1:	H T A	T E M	E R C	D I S	S I T	W H A
Scramble #2:	W E T	H R E	A C M	T S A	I I T	S D H
Scramble #3:	S T E	D M A	I T T	H A W	R E I	S C H
Scramble #4:	A H W	T M E	I A T	S T E	D H R	I S C

One of the scrambles was generated by randomly selecting the message letters from a hat while the other three scrambles were generated using an algorithm based on a basic geometrical concept.

Identify which is the random selection scramble and explain how/why you arrived at your decision.

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Scramble #2:	W E T	H R E	A C M	T S A	I I T	S D H
Scramble #3:	S T E	D M A	I T T	H A W	R E I	S C H
Scramble #4:	A H W	T M E	I A T	S T E	D H R	I S C

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Does this help?

#1	#2	#3	#4
H T A	W E T	S T E	A H W
T E M	H R E	D M A	T M E
E R C	A C M	I T T	I A T
D I S	T S A	H A W	S T E
S I T	I I T	R E I	D H R
W H A	S D H	S C H	I S C

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✓	CCSS.Math.Practice.MP4	**Model with mathematics.**
	CCSS.Math.Practice.MP5	**Use appropriate tools strategically.**
	CCSS.Math.Practice.MP6	**Attend to precision.**
✓	CCSS.Math.Practice.MP7	**Look for and make use of structure.**
✓	CCSS.Math.Practice.MP8	**Look for and express regularity in repeated reasoning.**

Connection/Extension Ideas

- How many total possible ways are there for scrambling the letters in the phrase?
- What are some modifications could you make to the scrambling algorithm?

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2

Explain why the modular scramble is not random.

Based on the modular scramble, what does the plaintext letter V get replaced with?

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2

Use numbers to define the modular scramble?

Use numbers to determine what the plaintext letter V get replaced with?

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2

Use numbers to define the modular scramble?

$$H \Rightarrow 8 \Rightarrow \underbrace{5 \cdot 8 = 40}_{\textit{multiplicative step}} \Rightarrow \underbrace{40 \equiv 14 \pmod{26}}_{\textit{equivalence step}} \Rightarrow 14 \Rightarrow N$$

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2

B L S M Y

We've received a secret word that has been enciphered using this modular scramble.

You're job is to decode this secret word.

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
Modular Scramble:	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B
	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕	⇕
	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2

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❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

$$\mathrm{L} \Rightarrow 12 \Rightarrow \underbrace{12 \div 5 = 2.4}_{division\ step} \Rightarrow \underbrace{2.4 \equiv eek!\ (\mathrm{mod}\, 26)}_{equivalence\ step} \Rightarrow ?? \Rightarrow ??$$

B L S M Y

We've received a secret word that has been enciphered using this modular scramble.

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❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

$$L \Rightarrow 12 \Rightarrow \underbrace{21 \cdot 12 = 252}_{\text{multiplicative step}} \Rightarrow \underbrace{252 \equiv 18 \pmod{26}}_{\text{equivalence step}} \Rightarrow 18 \Rightarrow R$$

$$\begin{aligned} 5 \bullet ? &\equiv 12 \pmod{26} \\ \underbrace{21 \bullet 5}_{\text{inverse}} \bullet ? &\equiv 21 \bullet 12 \pmod{26} \\ 1 \bullet ? &\equiv 252 \pmod{26} \\ ? &\equiv 18 \pmod{26} \end{aligned}$$

B L S M Y

We've received a secret word that has been enciphered using this modular scramble.

You're job is to decode this secret word.

❽ SNEAKY SUBSTITUTION

BIG IDEAS: Modular Arithmetic, Functions, Combinatorics, Going Backwards is Usually Harder

✓	CCSS.Math.Practice.MP1	**Make sense of problems and persevere in solving them.**
✓	CCSS.Math.Practice.MP2	**Reason abstractly and quantitatively.**
	CCSS.Math.Practice.MP3	**Construct viable arguments and critique the reasoning of others.**
✓	CCSS.Math.Practice.MP4	**Model with mathematics.**
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Connection/Extension Ideas

- How can you use matrix algebra in conjunction with modular arithmetic to encipher/decipher?
- Create your own cipher system!